# Extension of Emmons' Spot Theory to Flows on Blunt Bodies

KARL K. CHEN* AND NOEL A. THYSON†
*Avco Systems Division, Wilmington, Mass.*

**The transition region is considered to be characterized by the intermittent appearance of turbulent spots, which grow as they move downstream until they finally merge into one another to form the turbulent boundary layer. The intermittency factor for arbitrary axisymmetric body with zero angle of attack has been derived in an expression which can be reduced to the form of universal intermittency distribution of Dhawan and Narasimha in the case of straight tube or flat plate. A key factor to control flow conditions in the transition zone appears to be the spot formation rate, which has been deduced from the available data of the extent of transition zone. It was found that the spot formation rate depends not only on the transition Reynolds number but also on the Mach number. A comparison of the deduced spot formation rate with the neutral stability curves indicated that the neutral stability curves can be used as a guide to relate the spot formation rate to the transitional Reynolds number. Calculations of the transitional heat-transfer rate on a sphere in supersonic flow agree well with the experimental results.**

## Nomenclature

$a$ = velocity ratio of the upstream edge of a turbulent spot, cf. Eq. (8)
$b$ = velocity ratio of the downstream edge of a turbulent spot, cf. Eq. (8)
$G$ = spot formation rate parameter, cf. Eq. (8)
$g$ = source rate density, cf. Eq. (3)
$N$ = spot formation rate, cf. Eq. (6)
$q$ = local heat-transfer rate
$Re_{\Delta x}$ = Reynolds number based on the extent of transition zone
$Re_{tr}$ = Reynolds number based on the physical location of transition point
$Re_\theta$ = Reynolds number based on the momentum thickness
$r$ = body radius from the axis of symmetry
$s$ = streamwise surface distance measured from stagnation point
$t$ = time
$U_e$ = external velocity
$U_u$ = streamwise velocity of the upstream edge of a turbulent spot, cf. Fig. 1
$U_d$ = streamwise velocity of the downstream edge of a turbulent spot, cf. Fig. 1
$\alpha$ = angle between the locus of successive spot positions, cf. Fig. 1
$\beta$ = characteristic frequency, cf. Eq. (A1)
$\phi$ = azimuthal angle of an axisymmetric body
$\lambda$ = spanwise wave length, cf. Eq. (A1)
$\nu$ = kinematic viscosity
$\gamma$ = intermittency factor, cf. Eq. (3)

### *Subscripts*

0 = source point of a turbulent spot
$b$ = beginning of transition
1 = laminar
$s$ = stagnation
$t$ = turbulent
tr = transitional

## Introduction

AT low speeds, transition extends over a relatively small region. As the speed increases rapidly, the extent of transition zone in some cases becomes even larger than the laminar and turbulent regions, and the peak heating often occurs within or near the rear border of the transition zone.[1,2] Hence a reinvestigation of the model of the transition zone becomes necessary to predict the heating distribution and many other factors for the design of flight vehicles.

The transition region is usually considered to be characterized by the intermittent appearance of turbulent spots, which grow as they move downstream until they finally merge into one another to form the turbulent boundary layer. This concept was first discovered by Emmons[3] and then verified by the experiments of Schubauer and Klebanoff,[4] Elder[5] and many others. It was found that there are essentially no interaction effects between spots and that the intermittency factor at any point can be calculated by adding the effects of individual spots.

As the intermittency factor, which is defined as the fraction of time occupied by the turbulent spots, increases from 0 to 1 through the transition zone, all averaged properties adjust smoothly from laminar to turbulent boundary-layer values. Dhawan and Narasimha[6] proposed a universal intermittency distribution for boundary layers on a flat plate and then successfully predicted the skin-friction and velocity profiles in the transition region. Recently, Nagel[7] extended Emmons' transition theory to hypersonic flow and related the mean spot formation rate to the spanwise spacing of the peaks, which has been assumed to be proportional to the freestream turbulence scale. All of the previous works have been restricted to the cases of zero pressure gradient. The present analysis extends Emmons' spot theory for boundary layers on flat plates[3] or cones[8] to include those of blunt bodies. The spot formation rate, which is a key factor that effects flowfields in the transition region, can be deduced from the available data of the extent of transition zone. It was found that the formation rate of turbulent spots depends not only on the transition Reynolds number but also on the local Mach number. Finally, some calculations of the transitional heat-transfer rate based on the present model were compared with the experimental results.

## Intermittency

Let us consider the transition region of the boundary layer on an axisymmetric body with $r$ and $s$ denoting the radial distance from axis of symmetry and the distance along the surface measured from the stagnation point, respectively, while subscripts $b$ and $e$ indicate the beginning and ending of the transition region, respectively.

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* Staff Scientist. Member AIAA.
† Group Leader. Associate Member AIAA.

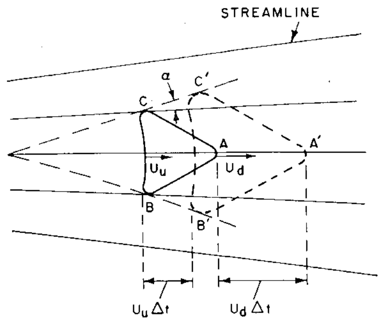


**Fig. 1 Spot geometry, growth and propagation.**

According to the experiments of Schubauer and Klebanoff,[4] Elder,[5] James,[9] etc., the turbulent spot can be approximated by a triangular shape with its characteristics described in Fig. 1, where $\alpha$ is the angle between the locus of successive spot positions and the local freestream line; $B$, $C$ and $U_u$ are the upstream edge tips and their common streamwise velocity, respectively; and $A$ and $U_d$ denote the downstream tip and its streamwise velocity, respectively.

Consider a spot moving in $s$, $\phi$, $t$ space where $s$, $\int r d\phi$, are coordinates on the body surface and $t$ is the time. A source at $P_0(s_0,\phi_0,t_0)$ produces a spot which sweeps out a volume whose shape depends upon the manner and rate of spot movement and growth. The locus of its downstream tip $A$, in $s,\phi,t$ space, is governed by‡ $\phi = \phi_0$ and

$$t - t_0 = \int_{s_0}^{s} U_d^{-1} ds \tag{1}$$

whereas those of upstream edge tips $B$ and $C$ are

$$\phi - \phi_0 = \int_{s_0}^{s} \tan\alpha / r ds \tag{2}$$

and

$$t - t_0 = \int_{s_0}^{s} U_u^{-1} ds$$

Under the assumption that turbulent spots appear randomly on the surface with a source rate density $g$, Emmons has shown that the intermittency factor at a given point $P$ is given by

$$\gamma = 1 - \exp\left( - \int_R g dV \right) \tag{3}$$

where $R$ is the influence volume of the point $P$ defined by the locus of all points $P_0$ which can influence the state of turbulence at point $P$. To make application of Eq. (3), it is necessary to know the source rate density $g$ and the influence volume $R$ for the resultant turbulent spot. The influence volume $R$ can be constructed through interchanging the constants and the variables of the equations describing the locus of the boundaries of the spot and then reversing the direction of time. For example, the locus of the source $P_0$ of the spot with its downstream tip at point $P(s,\phi,t)$ can be described by Eq. (2) with $(s,\phi,t)$ fixed and $(s_0,\phi_0,t_0)$ varying. Since the turbulent spots are triangular shaped, one would expect that the cross section of the influence volume at $s_0 = s'$ can also be approximated by a triangular with three tips determined from Eqs. (1) and (2). Thus the area of this cross section can be written as

$$A = r(s')\left[\int_{s'}^{s} \tan\alpha / r ds\right]\left[\int_{s'}^{s} (U_u^{-1} - U_d^{-1}) ds\right] \tag{4}$$

From Eqs. (3) and (4) the intermittency factor of the transitional boundary layer on an axisymmetric body with zero angle of attack can be expressed as follows:

$$\gamma = 1 - \exp\left\{ - \int_{s_0}^{s} g(s')r(s') \left[\int_{s'}^{s} \tan\alpha / r ds\right] \times \left[\int_{s'}^{s} (U_u^{-1} - U_d^{-1}) ds\right] ds' \right\} \tag{5}$$

where we have assumed the source rate density $g$ to be a function of $s'$ because of the axisymmetric consideration.

For the transitional boundary layer on a flat plate, Schubauer and Klebanoff[4] fitted their data of $\gamma$ with a Gaussian integral curve. Narasimha[11] tried to explain this by assuming that all the point-like breakdowns occur randomly across a single line $s = s_b$, or in other words that $g(s)$ can be approximated by a Dirac's delta function, which was reconfirmed by a later paper of Dhawan and Narasimha.[6] As a first approximation, we would like to accept this assumption that

$$g(s) = N\delta(s - s_b) \tag{6}$$

where $N$ denotes the spot formation rate at the line $s = s_b$ and will be discussed later.

We would like to take $\alpha$, $U_u/U_e$, $U_d/U_e$ to be constant as was found by Schubauer and Klebanoff[4] for subsonic flows on a flat plate and by James[9] for supersonic flows on straight tubes. Then Eq. (5) can be written as

$$\gamma = 1 - \exp\left\{-Gr(s_b) \left[\int_{s_b}^{s} r^{-1} ds\right]\left[\int_{s_b}^{s} U_e^{-1} ds\right]\right\} \tag{7}$$

where $G$ is a new defined spot formation rate parameter,

$$G = N \tan\alpha (a^{-1} - b^{-1}) \tag{8}$$

with

$$a = U_u/U_e \quad \text{and} \quad b = U_d/U_e$$

From Eq. (7), we may list the intermittency results for some simple shapes as follows: for a straight tube or flat plate, we have

$$\gamma = 1 - \exp\{-G(s - s_b)^2/U_e\} \tag{9}§$$

for cone,

$$\gamma = 1 - \exp-\{Gs_b[\ln(s/s_b)][(s - s_b)/U_e]\} \tag{10}$$

and for a hemisphere of radius $R$,

$$= 1 - \exp\Big(-G \sin(s_b/R)\{\ln[\tan(s/2R)/\tan(s_b/2R)]\} \times (\ln[s/s_b])/[R(dU_e/ds)_s]\Big) \tag{11}$$

where we have approximated the velocity to grow linearly from the stagnation point with subscript $s$ denoting stagnation-point conditions.

## Spot Formation Rate

The most sensitive behavior of spots to influence flow conditions in the transition zone appears to be the spot formation rate. However, there are very few measurements, even for the flat plate case, of the frequency of spot production. The observations by James,[9] through the spark shadowgraphs of small gun-launched model in free flight, clearly indicate that the spot production rate increases with increasing Reynolds number, but no quantitative Reynolds number dependence can be concluded from his observations. The present approach is to deduce the spot formation rate through a rather indirect but more reliable way, i.e., from the well-established data of the extent of transition zone under different physical conditions. Dhawan and Narasimha[6] indicated a relationship between the Reynolds number based on the extent of transition zone, $Re_{\Delta x}$, and the Reynolds number at transition point,

‡ The readers are suggested to refer to Figs. 6 and 7 of Ref. 3 with their $x$ and $y$ equivalent to our $s$ and $\int r d\phi$, respectively.

§ It is interesting to notice that Eq. (9) is identical to the form of universal intermittency distribution of Dhawan and Narasimha.[6]

| REFERANCES | MACH NO. | SYM. |
|---|---|---|
| SMITH & CLUTTER (20) | 0 | ○ |
| SCHUBAUER & SKRAMSTAD (17) | 0 | ■ |
| BENNETT (21) | 0 | ◇ |
| SILVERSTEIN & BECKER (22) | 0 | △ |
| SCHUBAUER & KLEBANOFF (4) | 0 | ▽ |
| DHAWAN & NARASIMHA (6) | 0 | ○ |
| COLES (23) | 2 | ⊵ |
| POTTER & WHITFIELD (10) | 2.5 | ▲ |
| | 3.5 | ◆ |
| | 4.5 | □ |
| | 5.0 | ● |

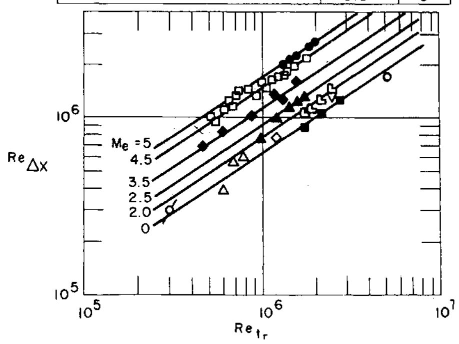


**Fig. 2 Dependence of transition extent on transition Reynolds number.**

$Re_{tr}$. More recently, Potter and Whitfield[10] have shown that the "transition extent Reynolds number" depends not only on the local Reynolds number at the transition point but also on the freestream Mach number. Figure 2 shows a plot of $Re_{\Delta x}$ vs $Re_{tr}$ for the available experimental data at various freestream Mach number. All the data selected are under conditions of nearly adiabatic wall. It is indicated that the effect of cooling to the transition extent Reynolds number needs further investigation. For rough estimations, mean curves can be used to indicate the dependence of $Re_{\Delta x}$ on $Re_{tr}$ for corresponding Mach number as shown in Fig. 2. The data are represented on the average by the equation

$$Re_{\Delta x} = A Re_{tr}^{0.67} \tag{12}$$

with $A$ as a function of Mach number. The dependence of $A$ on Mach number is shown in Fig. 3, and again the average can be represented by the following equation,¶

$$A = 60 + 4.68 M_e^{1.92} \tag{13}$$

If the transition zone is roughly defined as the region where the intermittency is smaller than 0.95 but larger than zero, the spot formation rate can be deduced from Eq. (8) through the substitution of Eqs. (12) and (13). Thus, we have

$$G\nu^2/U_e^3 = 3.0 Re_{tr}^{-1.34}/A^2 \tag{14}$$

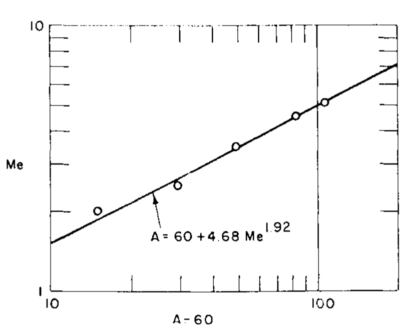


**Fig. 3 *A* vs Mach number.**

¶ For hypersonic flow, a recent paper of Mack[18] has indicated the increase of number of possible modes of instability with increasing Mach number. Those higher modes of instability could dominate the mechanisms of transition for high Mach number cases, say $M > 5$. Thus, Eqs. (13) and (15) are restricted to Mach number smaller than 5.

| REF. | Me | SYM |
|---|---|---|
| STETSON & RUSHTON [27] | 4.82 | ○ |
| WINTER ET. AL [6] | 2.45-3.25 | △ |
| EQ. (15) | | —— |

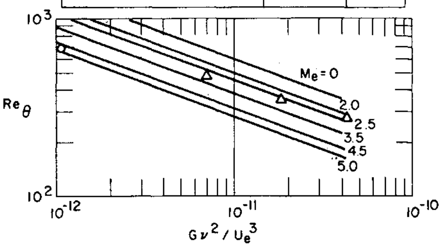


**Fig. 4 Comparison of Eq. (15) with data on cone.**

In view of the consideration that much of the transition phenomena is actually related to the $Re_{\theta_b}$, the Reynolds number based on the momentum thickness at the beginning of transition region, the formation rate parameter can be expressed in terms of $Re_{\theta_b}$ as follows:

$$G\nu^2/U_e^3 = Re_{\theta_b}^{-2.68}/A^2 \tag{15}$$

Where we have used the relationship, $\theta = 0.664\ (\nu x/U_e)^{1/2}$, derived from the laminar boundary-layer theory.[12] The applicability of Eq. (15) to flows with divergent free streamline was verified in Fig. 4, which shows the comparison of Eq. (15) with the spot formation rate deduced directly from the data of conical flow. The mechanisms of spot formation have been discussed in the Appendix, and it was concluded that Eq. (15) can be applied safely to flows over surfaces of convex curvature under moderate favorable pressure gradient. Thus one would expect that Eqs. (7, 13, and 15) can be used to determine the intermittency factor for the transitional boundary layers on blunt bodies with reasonable accuracy.

## Transitional Heat Transfer

As long as the intermittency factor $\gamma$ in the transition region has been accurately described, the mean local heat-transfer rate during transition is given by[6]

$$q_{tr} = q_l(1 - \gamma) + q_t\gamma \tag{16}$$

where $q_l$ and $q_t$ are the corresponding local heat-transfer rate for laminar and turbulent flow, respectively. The comparison of the theoretical predictions of the local heat-transfer rate during transition with experimental results is shown in Fig. 5 where the data were taken from recent Avco measurements of heat-transfer rate on a hemisphere at Mach number 5 (Ref. 24). The intermittency factor $\gamma$ of present model was evaluated from Eqs. (11, 13, and 15), whereas that of flat

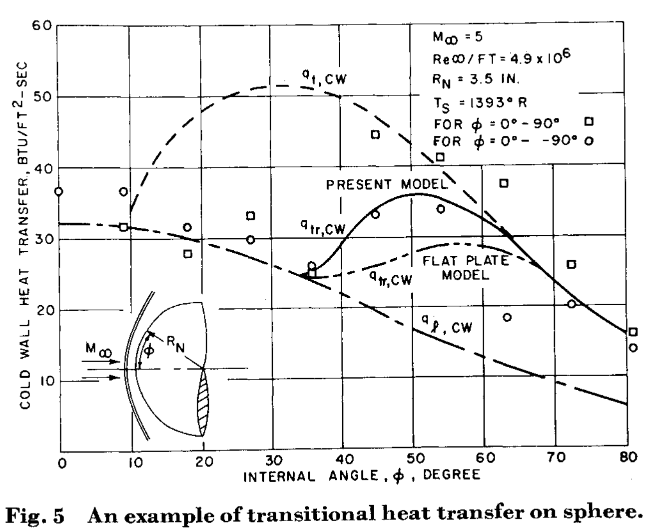


**Fig. 5 An example of transitional heat transfer on sphere.**

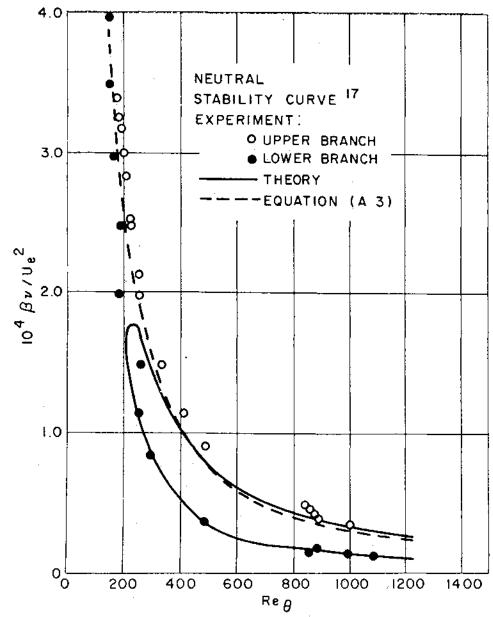


**Fig. 6 Comparison of Eq. (A3) with the neutral stability curve at subsonic flow.**

plate model was based on the universal intermittency distribution of Dhawan and Narasimha.[6] The laminar heat-transfer rate $q_l$ and the turbulent heat transfer $q_t$ were based on the local similarity solutions of Kemp, Rose and Detra[25] and on the method of Vaglio-Laurin,[26] respectively, with the subscript *cw* denoting the corresponding cold wall conditions. The comparison between the theory and experiments indicated that the present model gives good agreement for the transitional heat-transfer rate on blunt bodies.

## Concluding Remarks

The intermittency factor for arbitrary axisymmetric body with zero angle of attack has been derived in an expression that can be reduced to the form of universal intermittency distribution of Dhawan and Narasimha in the case of a flat plate or straight tube. The present model can be extended to cases of three-dimensional boundary layers if one assumes that the turbulent spots follow the corresponding inviscid streamlines at the outer edge of boundary layers and that the size of a single spot is rather small compared with the length scale to measure the variations of flow conditions along the body surfaces normal to the inviscid streamlines. In this case, the intermittency factor along a streamline can be described by Eq. (7) of the present paper through the substitution of metric coefficient $e_2$ for the body radius $r$. According to the present model, the formation rate of turbulent spots could be deduced from the well-established data of the extent of transition zone and was found to be dependent not only on the transitional Reynolds number but also on the Mach number. The comparison between the deduced spot formation rate and the neutral stability curve in the Appendix tends to support the conventional concept about the sequence of transition process, i.e., Tollmien-Schlichting waves, three-dimensional wave amplification, spanwise peak-valley development, breakdown, and turbulent spot development. The characteristic frequency of the disturbance spectrum would affect not only the location of transition point but also the frequency of spot formation—and so the extent of transition zone.

With the intermittency factor known through the transition zone, most of the averaged properties, such as velocity profile, skin friction, and heat-transfer rate, can be calculated according to Ref. 6.

## Appendix

In the experiments with controlled disturbances, Klebanoff, Tidstrom, Sargent and many others (e.g., Refs. 13 and 14) found that the Tollmien-Schlichting wave motions become strongly three-dimensional prior to transition with regularly spaced "peaks" and "valleys" in wave amplitude. The initial breakdown of laminar flow, which occurs only at the peaks, grows in succession once for each cycle of the primary oscillation. The local regions of breakdown are the initial stages in the development of turbulent spots. Thus the spot formation rate $N$ can be related with the frequency of the input disturbance as follows:

$$N = \beta/2\pi\lambda \tag{A1}$$

where $\beta$ is the frequency of controlled disturbance and $\lambda$ is the spanwise wave length.

In case of natural transition, which is initiated by a spectrum of disturbances such as existed in a wind tunnel, spanwise variations with "peaks" and "valleys" are still present, although less regularly spaced than those of controlled experiments.[15] The subsequent breakdown and so the turbulent spots are also found to be more numerous at the spanwise positions corresponding to the peak.

Recently, Reshotko[16] suggested the concept of characteristic frequency of the disturbance spectrum and showed the importance of its role in transition prediction. The existence of some correlation between the transition Reynolds number and the extent of transition zone as shown in Fig. 2 suggests that some kind of common factors, most likely the characteristic frequency, characterize both the transition Reynolds number and the spot production rate that is a key factor affecting the extent of transition zone. Thus, Eq. (A1) would be expected to be valid even for natural transition with $\beta$ denoting the characteristic frequency and $\lambda$ denoting some length scale equivalent to the preferred spanwise wave length.

So far there are no available data that can be used to estimate directly the values of $\beta$ or $\lambda$ under different test conditions of a given facility. However, since most spots are produced at "peak" region and overlap each other at the fully turbulent region, the average value of $\lambda$ can be approximated by the average width, of the spots when it begins to merge into each other, i.e.,

$$\lambda \approx a'\Delta x \tan\alpha \tag{A2}$$

where $\Delta x$ is the extent of transition zone. The wave length parameter $a'$ appears to be smaller than 2 and probably close to 1 from the consideration of the overlapping of spots after traveling a distance of $\Delta x$. Substitution of Eqs. (12, 15, and

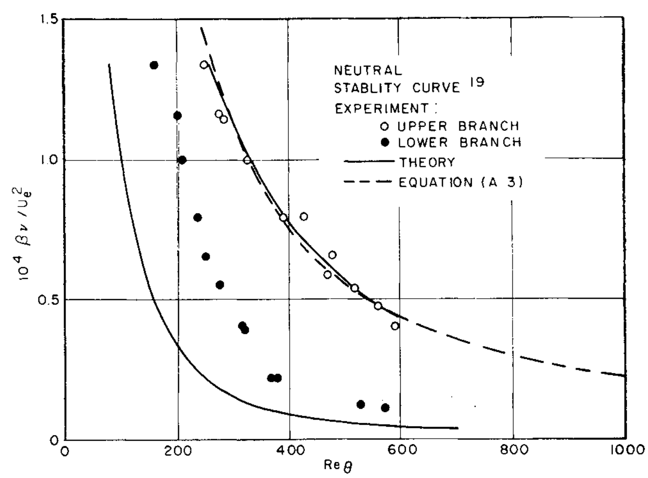


**Fig. 7 Comparison of Eq. (A3) with the neutral stability curve at $M_e$ = 2.2.**

A2) into (A1) yields

$$\beta\nu/U_e^2 = 16.3(a^{-1} - b^{-1})^{-1}Re_{\theta_b}{}^{-1.34}/A \qquad \text{(A3)}$$

where we take $a' = 1.5$, the average value between 1 and 2. Comparisons of Eq. (A3) with the neutral stability curves are shown in Fig. 7 for Mach number of 2.2 and in Fig. 6 for subsonic flow,** where the constants $a$ and $b$ are equal to 0.5 and 0.88, respectively, according to the experimental results of Ref. 4. The fact that the curve of Eq. (A3) is so close to the branch II of the neutral stability curves again supports the idea that the Tollmien-Schlichting waves play an important role in the process of natural transition; neutral stability curves can be used as a guide to relate the characteristic frequency $\beta$, and so the spot formation rate, to the location of transition point.

In order to get a feeling about the validity of Eq. (14) to flowfields ln blunt bodies, the pressure gradient effects and the surface curvature effects on the neutral stability curves would be discussed here. The experiments of Schubauer and Skramstad[17] indicated that the moderate favorable pressure gradient has no noticeable effect on the neutral stability curves. And a recent review paper by Tani[15] also made the statement that the Tollmien-Schlichting waves are expected to behave almost the same on a concave or convex surface as on a flat plate, but the flow on a concave surface exhibits another instability of Goertler vortex due to centrifugal pressure gradient. In any case, the situation of a flowfield on blunt bodies, which is under favorable pressure gradient and of convex surface, would not be expected to have noticeable effects on the neutral stability curves. With the neutral stability curves used as a guide relating spot formation rate to transition Reynolds number, the validity of Eq. (15) to flowfields on blunt bodies is supported.

** It is interesting to notice that the displacement of neutral stability curve in $Re_\theta$ due to compressibility effect agrees with that of Eq. (A3).

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